

This article was downloaded by:

On: 25 January 2011

Access details: *Access Details: Free Access*

Publisher *Taylor & Francis*

Informa Ltd Registered in England and Wales Registered Number: 1072954 Registered office: Mortimer House, 37-41 Mortimer Street, London W1T 3JH, UK



Separation Science and Technology

Publication details, including instructions for authors and subscription information:

<http://www.informaworld.com/smpp/title~content=t713708471>

Potential Applications of Pulsed Flow for Minimizing Concentration Polarization in Ultrafiltration

Shamsuddin Ilias^a; Rakesh Govind^a

^a Department of Chemical Engineering, University of Cincinnati, Cincinnati, Ohio

To cite this Article Ilias, Shamsuddin and Govind, Rakesh(1990) 'Potential Applications of Pulsed Flow for Minimizing Concentration Polarization in Ultrafiltration', *Separation Science and Technology*, 25: 13, 1307 – 1324

To link to this Article: DOI: 10.1080/01496399008050393

URL: <http://dx.doi.org/10.1080/01496399008050393>

PLEASE SCROLL DOWN FOR ARTICLE

Full terms and conditions of use: <http://www.informaworld.com/terms-and-conditions-of-access.pdf>

This article may be used for research, teaching and private study purposes. Any substantial or systematic reproduction, re-distribution, re-selling, loan or sub-licensing, systematic supply or distribution in any form to anyone is expressly forbidden.

The publisher does not give any warranty express or implied or make any representation that the contents will be complete or accurate or up to date. The accuracy of any instructions, formulae and drug doses should be independently verified with primary sources. The publisher shall not be liable for any loss, actions, claims, proceedings, demand or costs or damages whatsoever or howsoever caused arising directly or indirectly in connection with or arising out of the use of this material.

POTENTIAL APPLICATIONS OF PULSED FLOW FOR MINIMIZING CONCENTRATION POLARIZATION IN ULTRAFILTRATION

Shamsuddin Ilias & Rakesh Govind
Department of Chemical Engineering
University of Cincinnati
Cincinnati, Ohio 45221

ABSTRACT

Various methods have been proposed to control or minimize concentration polarization and membrane fouling. A brief review of these alternatives is presented in this paper. Flow pulsation as a means of improving transmembrane flux has been studied experimentally by a few investigators. A mathematical model is developed to evaluate the performance of a tubular membrane module under oscillatory flow conditions. Besides the effect of osmotic pressure and axial pressure variation, the model considers the convective-diffusive mass transport without decoupling the momentum equation from the solute continuity equation. Model equations are solved by a finite difference method as part of an iterative solution. Model predictions of transmembrane flux with experimental data are found to be in good agreement. By flow pulsing, it is possible to improve the transmembrane flux by more than 68% at pulsing frequency of $60 \text{ cycles} \cdot \text{min}^{-1}$. An analysis of extra power requirement for flow oscillation shows that the gain in transmembrane flux outweighs the cost of extra power, which is a minute fraction of the power required to maintain steady flow.

INTRODUCTION

Concentration polarization (CP) and membrane fouling are major concerns in the successful operation of membrane based separation processes, as their net effect is to reduce the permeate flux, thereby resulting in loss of productivity [1]. In many commercial plants, the transmembrane flux is limited by concentration polarization and membrane fouling. The flux may be as low as 2-10% of that

of pure water flux. Therefore, there is tremendous potential to make some efforts in order to reduce or control the polarization and fouling in membrane processes.

Flux decline due to membrane fouling are frequently confused with flux lowering phenomena associated with changes in membrane properties, changes in feed solution, or development of concentration polarization. Concentration polarization results in a localized increase in solute concentration on or near the membrane surface. This lowers the flux due to an increase in hydrodynamic resistance in the boundary layer and also, due to high local osmotic pressure resulting in a decreased driving force. However, concentration polarization effects are reversible, since its effects can be reduced by decreasing the transmembrane pressure or lowering the feed concentration. Fouling effects on the other hand, are characterized by an irreversible decline in flux. Changes in fluid management techniques may only increase the flux temporarily or mask the decline for a short period. The common practice to offset the fouling effects, is to shut down the process and clean the membrane by some chemical or other physical means [2,3].

In this paper, we briefly review the various methods and concepts that are currently being used or proposed to combat concentration polarization and fouling. Then we present a new model which employs the advantages of pulsed flow to improve the transmembrane flux in membrane separation processes.

BACKGROUND

There are at least three possible methods that can be used to reduce or control concentration polarization and fouling. These are: (1) changes in surface characteristics of the membrane, (2) feed treatment, and (3) modifications of flow rates and turbulence. Of these, the most preferred route would be modification of membrane chemistry so that attractive or adsorption forces are minimized [4,5]. However, this method requires the membrane to be customized for the constituents in each feed stream, which is not very realistic. The other possibilities by which membrane properties could be changed are by the use of a protective pre-coat, or by inducing a small electric current and in biotechnology, by immobilization of enzymes on the membrane surface [6-8]. The modification of membrane properties may help the reduction in flux decline phenomena; however this approach can not be viewed as a generic solution to the concentration polarization and fouling problem.

In Figure 1, we present a morphological analysis of ways to fight concentration polarization and fouling so as to improve the transmembrane fluxes [9]. All these options are based on three basic strategies. These strategies are: (1) low transmembrane flux, (2) low solid concentration in the feed streams, and (3) low concentration gradient between the membrane surface and the bulk of the liquid. For any practical purpose, the second strategy is of limited use, since it is not a practical solution to the problem.

Earlier development of membrane processes for commercial and pilot plant

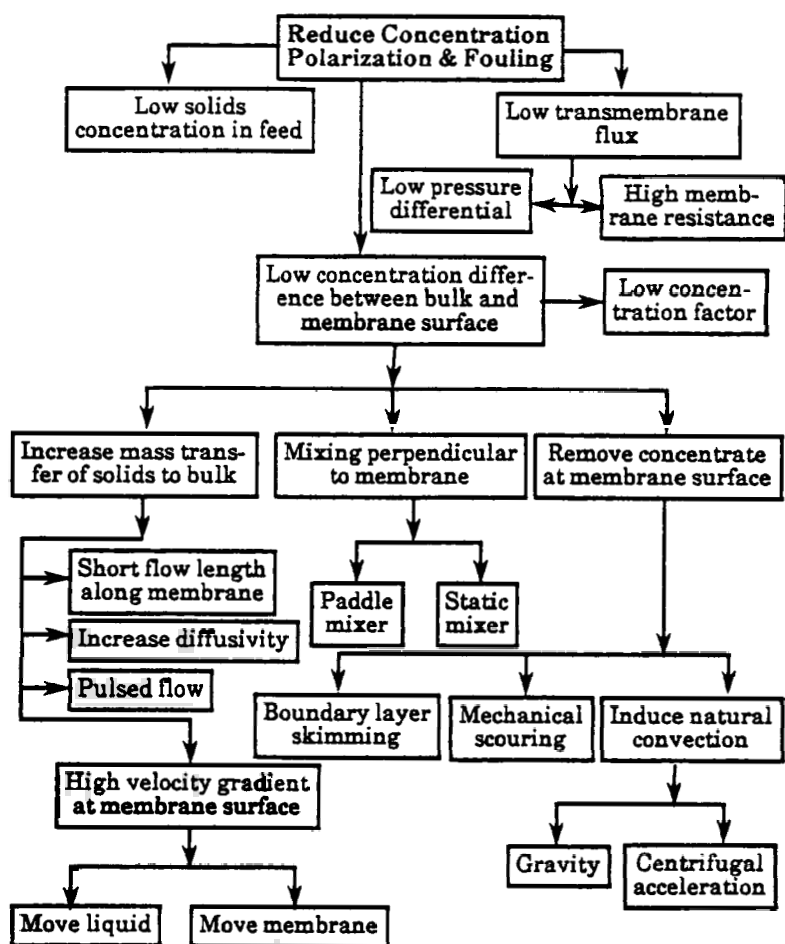


Figure 1: Approaches to reducing the effects of concentration polarization and membrane fouling.

applications were based on the strategy of maintaining low transmembrane fluxes. This is only possible in reasonable module capacity, if an extremely large membrane surface can be accommodated into a compact module. This resulted in the design of stacks of covered plates (plate-and-frame type) membrane modules by such manufactures as Dorr-Oliver, Millipore, and DDS (De Danske Sukkerfabrikker) and spiral wound configurations as manufactured by Abcor, and Osmonics. Another approach is to use bundles of hollow fiber membranes in tubular modules like Romicon, Asahi Kasei, and Amicon modules [3,10]. In all of these systems, where handling of feed containing suspended particles

or colloids and large macromolecules of high molecular weight (MW), feed pretreatment in the form of prefiltration is required to reduce plugging or fouling of the module. In these designs, no special effort is made to reduce or control concentration polarization and fouling other than manufacturers' specified operating conditions for a given system.

To combat concentration polarization, some researchers considered various possibilities of lowering concentration gradient between the bulk and the membrane surface. To achieve this, mixing can be done near the membrane surface by using paddle mixers. This is only feasible in small scale laboratory filtration modules but impractical for industrial application. Several investigators considered the use of static mixing devices in tubular modules as turbulence promoters. Experiments have shown that compared to empty feed channels, turbulent promoter often give higher permeate flux for the same feed velocity [11-14]. It is not clear whether this is caused by decrease in fouling due to removal of deposit from the membrane surface or it is caused by the reduction of the concentration boundary layer. However, static mixers (e.g., Kenix mixer) are only suitable for larger tubular designs.

To improve the transmembrane flux, several researchers considered the removal of fouling deposit or concentrate at the membrane surface by mechanical means [15,16], by induction of natural convection current [17-19] or by boundary layer skimming [20]. In mechanical cleaning, 'scouring balls' in large tubular membrane modules have been studied by several investigators. Regular cleaning of tubular membrane modules by pumping tight-fitting sponges through the system at fixed time intervals have been applied frequently. This is at best considered as a preventive maintenance of the system. Lowe et al [15] developed stacked filled modules with mechanical cleaning balls, which is known as the WURSTACK system. The cleaning action is obtained by rolling balls in the channels between the membranes. Scouring action of the balls improved the transmembrane flux considerably.

Lee and Lightfoot [20] proposed the concept of boundary layer skimming. This concept is simple and supports the old saying that "if you cannot beat them, join them". When concentration polarization and fouling is a problem, and if the desired product is the retentate (not the permeate), boundary layer skimming implies that the productivity could be increased substantially by not fighting but removing the concentrate from the boundary layer. However, commercial exploitation of this concept is in doubt because of unusual design requirements.

Another approach is to induce natural convection in the vicinity of the membrane and this idea drew attention from a number of investigators. High solids concentration at the membrane surface often results in higher liquid density than in the bulk liquid. Thus, buoyancy forces in a gravitational and centrifugal force field can stimulate liquid motion along the membrane surface or away from the membrane depending upon the physical arrangement of the system with the force fields [17-19]. From laboratory experiments, it has been shown that these concepts can be used to reduce concentration polarization.

However, any design based on these concepts, will require extra energy for rotation. Furthermore, the complexity of the modules will probably inhibit commercial development of the rotary membrane modules.

In another development, some investigators considered the use of fluidized bed concept to operate tubular membrane modules in a vertical position [21,22]. Fluidizing particles can be either metal or glass balls. In this case the improved transmembrane flux that is achieved, is due to the combination action of scouring and radial mixing.

Different types of modules have been developed which are based on the principle that the membrane itself or the surface parallel to the membrane is rotating. In this way one can achieve flow conditions independent of the feed flow through the module. One of these rotating modules is made out of a pair of concentric cylinders where the inner one is rotating and carries the membrane. Under certain conditions, a so called 'Taylor vortex' can be formed in the annulus. This means that beside the high-shear flow, there exists a secondary flow consisting of regular toroidal vortices rotating inside the annulus. This can improve the mass transfer considerably [23-25]. However, like many other innovative approaches, energy consumption for rotation as well as complex engineering design will probably inhibit commercial development of this type of membrane module.

Kennedy et al [26] showed that mass transfer coefficient in reverse osmosis (RO) of sucrose solution can be increased by pulsing the feed flow over the membrane. In their RO experiments under laminar and turbulent flow conditions, they reported increase in permeation rates up to 70% at frequencies of $60 \text{ cycles} \cdot \text{minute}^{-1}$. In ultrafiltration and microfiltration of whey and whole blood, Bauser et al [8] reported 40% increase in flux at 60 and $30 \text{ cycles} \cdot \text{minute}^{-1}$ for whey and whole blood, respectively. These studies suggest that the use of pulsed or oscillatory flow superimposed on main flow can be advantageously used to increase the transmembrane flux. This would require the manipulation of amplitude and frequency of the oscillatory component in favor of reducing concentration polarization and fouling. In a recent article, Belfort [27] reviewed the role hydrodynamics in controlling membrane fouling and concentration polarization in membrane filtration.

From this review, it is clear that various innovative methods have been proposed to combat concentration polarization and fouling. These have been partially successful, and in many cases were found to be difficult to retrofit existing installations. In some cases modifications were difficult from engineering and economic design considerations. In this paper, following the experimental work of Kennedy et al [26], we present a mathematical model based on oscillatory flow in tubular membrane to demonstrate the advantages over conventional tubular flow modules. Also, from the consideration of excess energy for pulsed flow, we give an engineering analysis of this approach to justify its potential application in improving transmembrane flux in membrane processes.

MODEL DEVELOPMENT

We consider the flow of Newtonian fluid in a permeable tubular membrane where a periodic pressure gradient is imposed on the main flow. The flow is laminar, and the mass density, viscosity and solute diffusivity are assumed to be constant. The appropriate governing equations that describe the equation of continuity and momentum and continuity of solutes are [28]:

$$r \frac{\partial u}{\partial z} + \frac{\partial}{\partial r}(vr) = 0 \quad (1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial z} + v \frac{\partial u}{\partial r} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \frac{\nu}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) \quad (2)$$

$$\frac{\partial c}{\partial t} + u \frac{\partial c}{\partial z} + v \frac{\partial c}{\partial r} = \frac{D}{r} \frac{\partial}{\partial r} \left(r \frac{\partial c}{\partial r} \right) \quad (3)$$

where u and v are the axial and radial velocity components, c is the solute concentration, ρ and ν are the density and kinematic viscosity of the fluid, respectively and D is the solute diffusivity.

Following the analysis of Edwards and Wilkinson [29], we assume $\partial p / \partial z$ has the following form:

$$\frac{\partial p}{\partial z} = \left(\frac{\partial p}{\partial z} \right)_s [1 + \varepsilon \exp(i\omega t)] \quad (4)$$

Here ε is the ratio of the amplitude of the pressure gradient for oscillatory and steady flow. It can be shown that the oscillatory component of the imposed velocity is given by [30]:

$$u'(r, t) = \frac{B_0 M'_0}{\rho \omega} \sin[\omega t + \Phi_0(r)] \quad (5)$$

where B_0 is the amplitude of oscillatory pressure gradient and the quantity M'_0 is defined as:

$$M'_0 = (1 + \xi_0^2 - 2\xi_0 \cos \Psi_0)^{1/2} \quad (6)$$

where

$$\xi_0 = \frac{M_0(\Omega R)}{M_0(\Omega)}; \text{ and } \Psi_0 = \Theta_0(\Omega) - \Theta_0(\Omega R) \quad (7)$$

and the phase angle is given by

$$\Phi_0(r) = \tan^{-1} \frac{\xi_0 \sin \Psi_0}{1 - \xi_0 \cos \Psi_0} \quad (8)$$

with M_0 and Θ_0 as the modulus and phase of zero order Bessel function and Ω is the dimensionless frequency parameter. The velocity in this situation is obtained by summing the steady flow and oscillatory flow components [29].

Thus, if the usual parabolic velocity profile for steady flow is added to Equation (5), then we get,

$$u(r, t) = \frac{B_s r_i^2}{4\mu} \left(1 - \frac{r^2}{r_i^2}\right) + \frac{B_0 M'_0}{\rho\omega} \sin[\omega t + \Phi_0(r)] \quad (9)$$

Equation (9) now gives the inflow condition at the inlet of the tubular membrane module. The initial and boundary conditions for the system of Equations (1-3), are:

$$u(0, r, z) = u_s(0, r, z) \quad (10)$$

$$u(t, r, 0) = \frac{B_s r_i^2}{4\mu} \left(1 - \frac{r^2}{r_i^2}\right) + \frac{B_0 M'_0}{\rho\omega} \sin[\omega t + \Phi_0(r)] \quad (11)$$

$$c(0, r, z) = c_s(0, r, z); \quad c(t, r, 0) = c_0 \quad (12)$$

$$\left. \frac{\partial u}{\partial r} \right|_{r=0} = 0; \quad \left. \frac{\partial c}{\partial r} \right|_{r=0} = 0 \quad (13)$$

$$v(t, r, z)|_{r=r_1} = v_w = A(\Delta p - \Delta \pi) \quad (14)$$

$$D \left. \frac{\partial c}{\partial r} \right|_{r=r_1} = \beta v_w c_w \quad (15)$$

where π is the osmotic pressure which depends on the solute concentration, β is the solute rejection coefficient of the membrane at membrane surface and A is the effective membrane permeability. The subscript w refers to the condition at the wall, the subscript s refers to the steady flow condition.

The boundary conditions are given in most general form, which take into account all possible cases, such as variable wall flux condition, effect of osmotic pressure, etc. In the absence of oscillatory velocity component, it represents the steady state case. The membrane permeability, A in Equation (14) is assumed to be constant. However, if one has to incorporate the effect of fouling, then the variation of membrane permeability with time has also to be known in some functional form. Noting that, Equations (14-15) are coupled by v_w and c_w , with proper model for membrane permeability under fouling condition, the present model can be easily used to describe the effect of fouling in concentration polarization. However, at this time there is no adequate model for membrane permeability under fouling condition, and hence, we will limit our present work to concentration polarization only. For generality, the model equations and the boundary conditions are written in dimensionless form as:

Governing Equations:

$$R \frac{\partial U}{\partial Z} + \frac{\partial}{\partial R}(VR) = 0 \quad (16)$$

$$\frac{\partial U}{\partial \tau} + U \frac{\partial U}{\partial Z} + V \frac{\partial U}{\partial R} = -\frac{1}{2} \frac{\partial P}{\partial Z} + \frac{1}{Re_{w_0} R} \frac{\partial}{\partial R} \left(R \frac{\partial U}{\partial R} \right) \quad (17)$$

$$\frac{\partial C}{\partial \tau} + U \frac{\partial C}{\partial Z} + V \frac{\partial C}{\partial R} = \frac{1}{Pe_0 R} \frac{\partial}{\partial R} \left(R \frac{\partial C}{\partial R} \right) \quad (18)$$

Initial and Boundary Conditions:

$$U(0, R, Z) = U_s(0, R, Z) \quad (19)$$

$$U(\tau, R, 0) = 2(1 - R^2) + \frac{B_0 M'_0}{u_0 \rho \omega} \sin[(\omega r_i / v_{w_0})\tau + \Phi_0(R)] \quad (20)$$

$$C(0, R, Z) = C_s(0, R, Z); \quad C(\tau, R, 0) = 1 \quad (21)$$

$$\left. \frac{\partial U}{\partial R} \right|_{R=0} = 0; \quad \left. \frac{\partial C}{\partial R} \right|_{R=0} = 0 \quad (22)$$

$$V(\tau, R, Z)|_{R=1} = V_w = \bar{A}(\Delta P - \Delta \Pi) \quad (23)$$

$$\left. \frac{\partial C}{\partial R} \right|_{R=1} = P c_0 \beta V_w C_w \quad (24)$$

The solution of Equations (16-18), subject to initial and boundary conditions, Equations (19-24) will enable one to analyze the performance of the membrane module for various parameters such as frequency and amplitude of oscillatory flow and other operational and physical conditions. It is to be noted that the above model equations and the boundary conditions are equally applicable to both UF and RO processes in cross flow tubular membrane modules.

METHOD OF SOLUTION

No analytical solution is known for the system of Equations (16-18), subject to above initial and boundary conditions. It is to be noted that Equations (23) and (24) give the coupled boundary conditions for wall flux and solute mass balance of convective-diffusive transport at the membrane surface with solute rejection. Thus, to solve the coupled model equations, one has to resort to some numerical methods. In this work, Equations (16-18) are solved by a finite difference method implicit in radial direction. A system of grid lines running in Z - and R - directions, i.e., i and j lines, are imposed on the solution domain. In the finite difference approximation, the convective terms $U \frac{\partial U}{\partial Z}$, $V \frac{\partial U}{\partial R}$, $U \frac{\partial C}{\partial Z}$, and $V \frac{\partial C}{\partial R}$ at interior grid point (i, j) were linearized by approximating the coefficients U and V at $(i - 1, j)$, i.e. by taking the known velocities at the previous grid point. The second order derivative terms were approximated by three-point centered difference scheme and the time derivatives were approximated by forward time difference scheme. Equation (16) was discretized by a centered difference scheme by taking the derivatives at $(i, j - 1/2)$. The derivative boundary conditions at the axis of symmetry were approximated by three-point forward difference, while, the derivative condition at the membrane surface was given by three-point backward difference formula. An iterative procedure was developed to solve the discretized governing equations with necessary boundary conditions. The basic algorithm is described elsewhere [31].

RESULTS AND DISCUSSION

In this paper, we considered the convection-diffusion mass transport in a tubular membrane module under oscillatory flow conditions. For numerical simulation, sucrose solution was taken as feed solution whose osmotic pressure as a function of concentration ($10\% \leq c \leq 26\%$) is given by [26]:

$$\pi = 1197.492c/(100 - c) - 11.79 \quad (25)$$

In absence of oscillatory flow, the performance of the membrane module under steady state condition was taken as a base case for a given set of operating conditions and it was used to compare the goodness of oscillatory flow mode for identical operating conditions.

Typical plots of the radial and axial variation of solute concentration and velocity components, for the base case, have been shown in Figures (2) and (3), respectively. Figure (2) shows the radial variation of dimensionless concentration, axial and radial velocities at axial location $Z/Z_{\max} = 0.5$. Due to loss of permeate, the axial velocity profile deviates from the fully developed parabolic flow. The transverse velocity profile develops from centerline to the membrane wall. Due to convection-diffusion, the solute concentration profile also develops from centerline to the membrane wall.

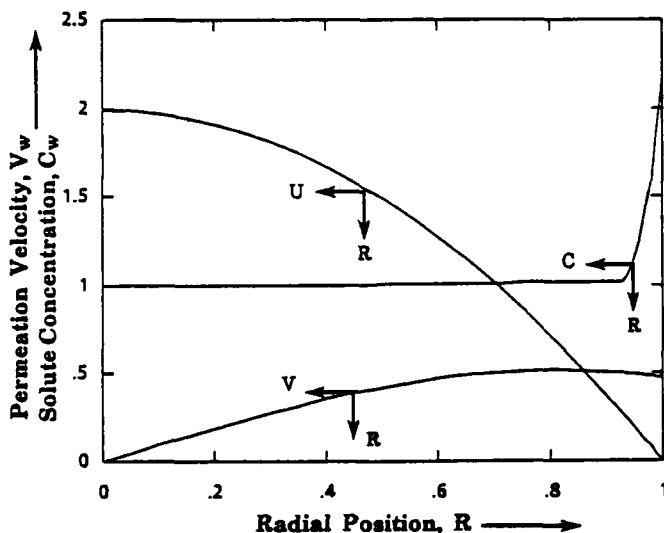


Figure 2: Radial variation of dimensionless axial velocity (U), transverse velocity (V), and solute concentration (C) at the axial location, $Z/Z_{\max} = 0.5$ under steady flow condition ($U_0 = 916.29 \text{ cm} \cdot \text{min}^{-1}$, and $A = 7.34 \times 10^{-5} \text{ cm} \cdot \text{psi}^{-1} \cdot \text{min}^{-1}$).

The axial variation of solute concentration at the membrane surface and wall permeation velocity are shown in Figure (3). The surface solute concen-

tration increases along the axial length. This results in an increase in osmotic pressure difference between the permeate and feed side. This increase in osmotic pressure reduces the transmembrane flux along the axial position, as shown in Figure (3).

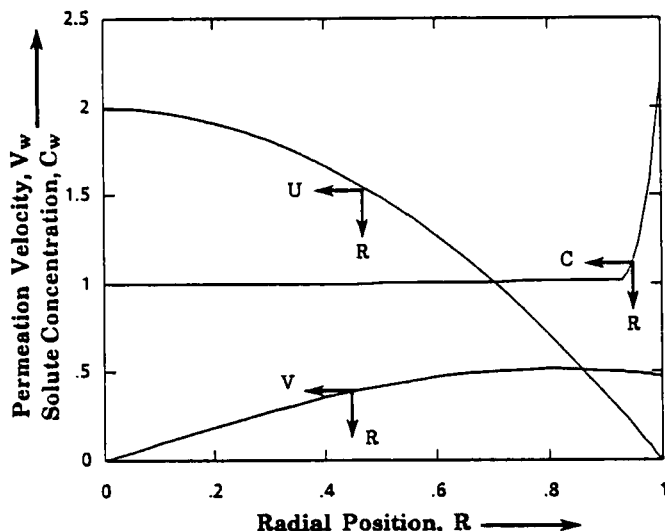


Figure 3: Axial variation of dimensionless wall permeation velocity (V_w), and solute concentration (C_w) at membrane surface under steady flow condition ($U_0 = 916.29 \text{ cm} \cdot \text{min}^{-1}$, and $A = 7.34 \times 10^{-5} \text{ cm} \cdot \text{psi}^{-1} \cdot \text{min}^{-1}$).

The effect of amplitude and frequency of oscillation on average wall permeate flux are shown in Figures (4) and (5). In Figure (4), for frequency, $f = 20 \text{ cycles} \cdot \text{min}^{-1}$, the variation of wall permeation rate with time is shown for three amplitudes, $a = 28, 18.67$, and 9.34 cm of membrane tube length. In Figure (5), $f = 60 \text{ cycles} \cdot \text{min}^{-1}$ has been used to show the effect of frequency on wall permeate flux. The steady state permeate flux in the absence of any flow pulsation, is shown by dotted lines (base case) in Figures (4) and (5). From Figures (4) and (5), it is clear that wall permeate flux increases with both frequency and amplitude. If we compare the enhancement in permeate flux under oscillatory flow condition with that of base case (steady state flow), we find that the flux increases up to 66.8%, 43.8% and 22.5% for amplitude $a = 28, 18.67$, and 9.4 cm , respectively with frequency of oscillation, $f = 60 \text{ cycles} \cdot \text{min}^{-1}$. At a lower frequency, $f = 20 \text{ cycles} \cdot \text{min}^{-1}$, the flux enhancement is about 20 %, 11.4% and 3.5% at amplitudes $a = 28, 18.67$ and 9.4 cm respectively. This enhancement in permeate flux is consistent with the experimental observation of Kennedy et al [26].

To validate the numerical simulation of oscillatory flow model, we considered the experimental data of Kennedy et al [26]. The module dimensions

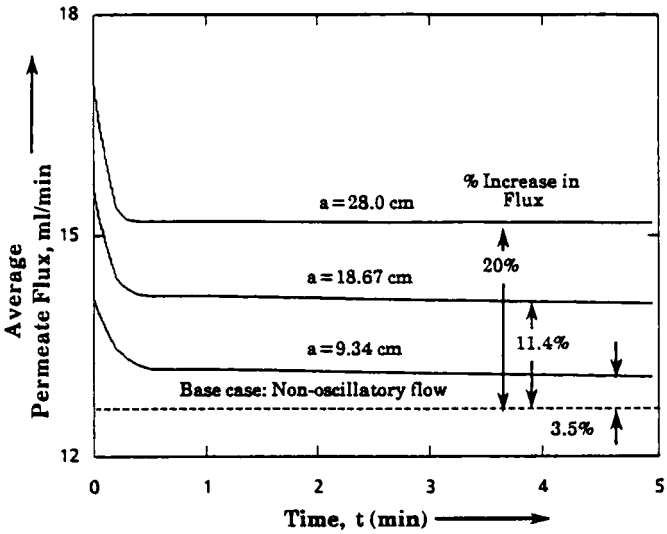


Figure 4: Average wall permeate flux as a function of time for various amplitudes with frequency, $f = 20 \text{ cycles} \cdot \text{min}^{-1}$, under oscillatory flow condition ($U_0 = 916.29 \text{ cm} \cdot \text{min}^{-1}$, and $A = 7.34 \times 10^{-5} \text{ cm} \cdot \text{psi}^{-1} \cdot \text{min}^{-1}$).

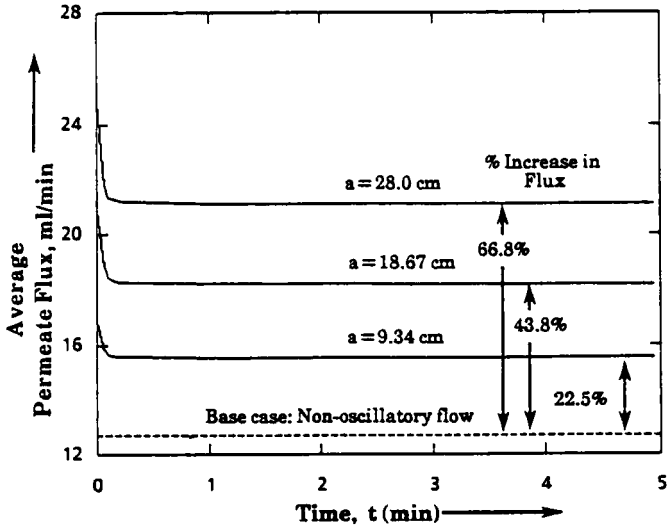


Figure 5: Average wall permeate flux as a function of time for various amplitudes with frequency, $f = 60 \text{ cycles} \cdot \text{min}^{-1}$, under oscillatory flow condition ($U_0 = 916.29 \text{ cm} \cdot \text{min}^{-1}$, and $A = 7.34 \times 10^{-5} \text{ cm} \cdot \text{psi}^{-1} \cdot \text{min}^{-1}$).

and other related parameters are listed in Table 1.

Table 1
Input Data and Physical Parameters [26]

Module dimension:	Length = 219 cm	Dia = 1.31 cm	
Membrane Material:	Cellulose Acetate		
Feed Solution:	10% w/w Sucrose		
Operating Pressure:	500 psig	Solute Rejection:	98 %
System Parameters	I	II	III
$A, \text{cm} \cdot \text{min}^{-1} \cdot \text{psi}^{-1}$	7.25×10^{-5}	6.87×10^{-5}	7.34×10^{-5}
a, cm	28.0	18.67	9.34
$f, \text{cycles} \cdot \text{min}^{-1}$	0-50	0-60	0-60
$u_0, \text{cm} \cdot \text{min}^{-1}$	916.29	901.45	897.74

In Figures (6) through (8), the experimental data of Kennedy et al [26], and their theoretical prediction are presented along with our model predictions. Following the analysis of Merten [32], Kennedy et al [26] expressed the permeation velocity and solute concentration at the membrane surface as:

$$v_w = A(\Delta p - \Delta \pi) \quad (26)$$

$$c_w = c_b e^{v_w/k} \quad (27)$$

From square wave theory [33], the ratio of the pulsed to steady flow mass transfer coefficients was approximated as:

$$\frac{k_0}{k_s} = \frac{1}{2} \left| 1 + \frac{4af}{u_0} \right|^n + \frac{1}{2} \left| 1 - \frac{4af}{u_0} \right|^n \quad (28)$$

where, $n = 1/3$ for laminar flow and $n = 0.8$ for fully developed turbulent flow. Using the experimental values of v_w and c_b , k_s was computed via Equations (26-28) by trial-and-error iteration. The bulk concentration c_b was taken as the average between the inlet and outlet concentrations. It can be seen from Figures (6-8), that the square wave theory fails to predict the permeation rate data adequately. It was argued that this failure may be due to negligence of natural convection in the development of model equations, Equations (27) and (28).

In our numerical simulation, the effects of convective-diffusive mass transport, axial variation of transmembrane pressure and osmotic pressure were considered without decoupling solute continuity equation from the momentum equation. The improvement in the prediction of permeation rate data is evident in Figures (6-8) when compared with that by the square wave theory.

From this theoretical study, it is now clear that under oscillatory flow conditions, the transmembrane flux can be substantially increased by manipulating the frequency and amplitude of oscillations. To advocate this mode of operation over other traditional systems, one has to show its advantages from the cost-benefit analysis of this system along with other competing alternatives. To implement the oscillatory flow mode in existing membrane modules,

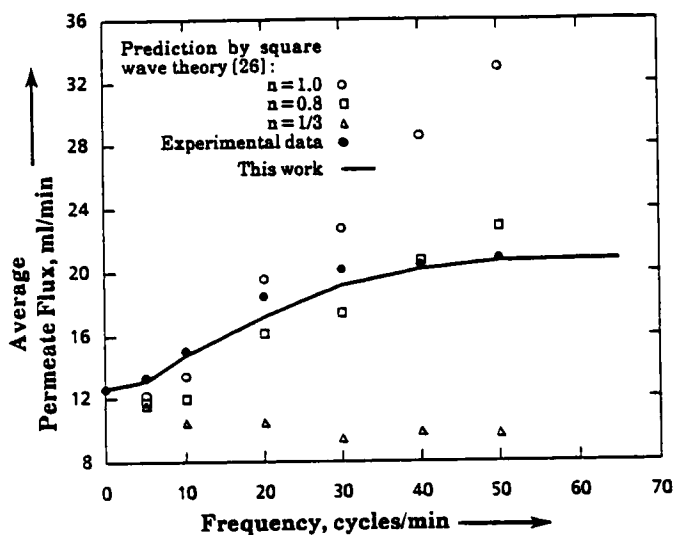


Figure 6: Comparison of the experimental permeation rates [26] with model predictions at various frequencies ($U_0 = 916.29 \text{ cm} \cdot \text{min}^{-1}$, $a = 28.0 \text{ cm}$ and $A = 7.34 \times 10^{-5} \text{ cm} \cdot \text{psi}^{-1} \cdot \text{min}^{-1}$).

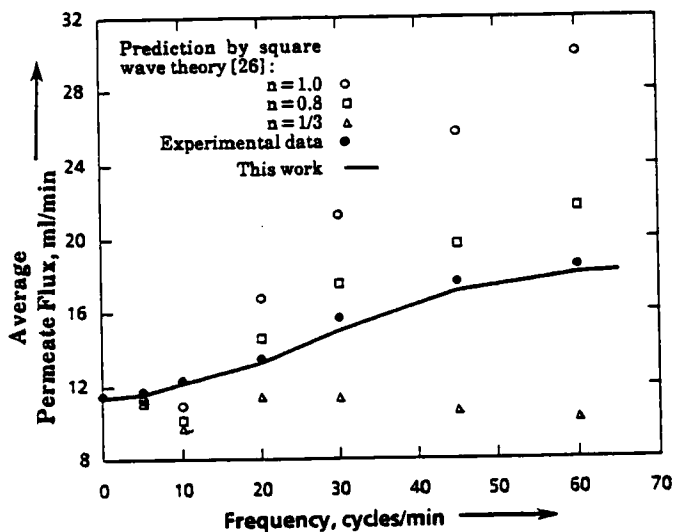


Figure 7: Comparison of the experimental permeation rates [26] with model predictions at various frequencies ($U_0 = 901.45 \text{ cm} \cdot \text{min}^{-1}$, $a = 18.67 \text{ cm}$ and $A = 6.87 \times 10^{-5} \text{ cm} \cdot \text{psi}^{-1} \cdot \text{min}^{-1}$).

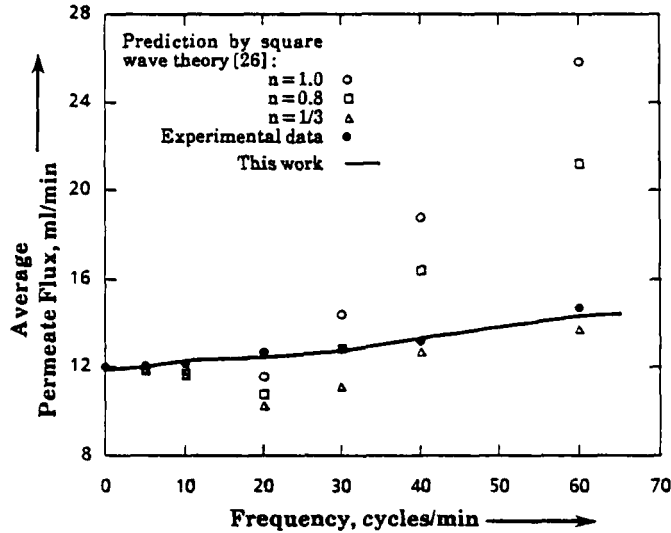


Figure 8: Comparison of the experimental permeation rates [26] with model predictions at various frequencies ($U_0 = 897.74 \text{ cm} \cdot \text{min}^{-1}$, $a = 9.43 \text{ cm}$ and $A = 7.34 \times 10^{-5} \text{ cm} \cdot \text{psi}^{-1} \cdot \text{min}^{-1}$).

one would only require a pumping device to generate pulsation with appropriate controls to operate under desired frequency and amplitude. For preliminary evaluation, it is important to evaluate the power requirement for this system to sustain the oscillatory flow.

Let us consider the instantaneous flow rate $Q(t)$ of a Newtonian fluid under oscillatory flow conditions, which is given for flow in a tube as[28]:

$$Q(t) = 2\pi \int_0^r u(r, t) r \, dr \tag{29}$$

where $u(r, t)$ is given by Equation (9). The above integral can be evaluated and expressed in the form:

$$Q(t) = Q_s + Q_0 \sin(\omega t + \Phi_1) \tag{30}$$

where

$$Q_s = \frac{\pi r_i^2}{8\mu} B_s \tag{31}$$

and

$$Q_0 = \frac{\pi r_i^4 M'_{10}}{\mu \Omega^2} B_0 \tag{32}$$

The quantity M'_{10} is defined as [29]:

$$M'_{10} = \frac{2M_1(\Omega)}{\Omega M_0(\Omega)} \left\{ \sin^2 \sigma + \left[\frac{\Omega M_0(\Omega)}{2M_1(\Omega)} - \cos \sigma \right]^2 \right\}^{1/2} \quad (33)$$

with

$$\sigma = 135^\circ - \Theta_1(\Omega) + \Theta_0(\Omega) \quad (34)$$

where M_1 and Θ_1 are the modulus and phase of 1st order Bessel function. The phase lag Φ_1 is given by:

$$\Phi_1 = \tan^{-1} \frac{\sin \sigma}{\frac{\Omega M_0(\Omega)}{2M_1(\Omega)} - \cos \sigma} \quad (35)$$

The time average flowrate $\bar{Q}$ is given by:

$$\bar{Q} = \frac{\omega}{2\pi} \int_0^{2\pi/\omega} Q(t) dt \quad (36)$$

It is to be noted that $\bar{Q}$ is the effective delivery under oscillatory flow conditions. Integration of Equation (36) shows that the mean flowrate, $\bar{Q}$ is not dependent upon the pulsations, i.e., $\bar{Q}$ is always equal to the Q_s under steady pressure gradient B_s . Thus, flow oscillations do not increase the delivery rate for Newtonian fluids. With this, it is now possible to calculate the power requirement per unit length, $\mathcal{P}$ to sustain the oscillatory flow, which is given by:

$$\mathcal{P} = \frac{\omega}{2\pi} \int_0^{2\pi/\omega} Q(t) \frac{\partial p}{\partial z} dt \quad (37)$$

where $\frac{\partial p}{\partial z}$ is given by Equation (4). If we integrate Equation (37), the power requirement, $\mathcal{P}$ can be shown to be given by:

$$\mathcal{P} = \frac{\pi r_1^4}{8\mu} B_s^2 \left[1 + \varepsilon^2 F(\Omega) \right] \quad (38)$$

where $F(\Omega) = 4M'_{10} \sin \Phi_1 / \Omega^2$. The first term on the right hand side of Equation (38) is the power requirement to maintain the steady flow component Q_s , while the second term is that required to maintain the pulsating flow. Thus the term $\varepsilon^2 F(\Omega)$ may be viewed as excess power consumption coefficient.

Based on the above analysis, in Table 2, we present the excess power requirement for oscillatory flow at various amplitudes and frequency of oscillations. From Table 2, it can be seen that the excess power requirement for the oscillatory flow is insignificant and is always a minute fraction of the power that is required to maintain the steady-state flow. Thus, the oscillatory flow mode of operation presents a viable and attractive means to improve the transmembrane flux. The gain in flux would always outweigh the cost incurred due to excess power requirement. The pumping device that will be needed to implement the pulsating flow mode operation, if manufactured in large scale for commercial applications, would be cost effective.

Table 2

Excess power requirement for oscillatory flow and gain in transmembrane flux under various frequencies and amplitudes of oscillations.

Feed flowrate, u_0 , $\text{cm} \cdot \text{min}^{-1}$	Frequency, f cycles $\cdot \text{min}^{-1}$	Amplitude, a , cm	Excess Power Coeff., $\varepsilon^2 F(\Omega)$	Improvement in Flux, %
916.29	20	28.0	0.0002296	20.0
916.29	20	18.67	0.0001021	11.4
916.29	20	9.4	0.0000259	3.5
916.29	60	28.0	0.0003652	66.8
916.29	60	18.67	0.0001624	43.80
916.29	60	9.4	0.0000412	22.5

CONCLUSIONS

A mathematical model is presented to evaluate the performance of a tubular membrane module under oscillatory flow conditions. By flow pulsing, it is possible to improve the transmembrane flux substantially. Preliminary evaluation of extra power requirement for oscillatory flow shows that the gain in transmembrane flux outweighs the cost of extra power. Thus, the system offers an alternative, cost effective method to improve the transmembrane flux.

NOMENCLATURE

A	membrane permeability, $\text{cm} \cdot \text{min}^{-1} \cdot \text{psi}^{-1}$
$\bar{A}$	dimensionless membrane permeability, $A\rho u_0^2/2v_{w0}$
a	pulsed amplitude, cm of membrane tube length
B_s	steady pressure gradient
B_0	amplitude of oscillatory pressure gradient, $a\mu/(2r_i^2 M'_{10}/\Omega^2)$
c	concentration of solute, w%
c_b	solute concentration in the bulk, w%
c_w	surface solute or gel concentration, w%
c_0	inlet feed solute concentration, w%
C	dimensionless solute concentration, c/c_0
C_w	dimensionless surface solute concentration, c_w/c_0
D	solute diffusivity, $\text{cm}^2 \cdot \text{min}^{-1}$
f	pulse frequency, cycles $\cdot \text{min}^{-1}$
k	mass transfer coefficient, $\text{cm} \cdot \text{min}^{-1}$
k_0	mass transfer coefficient for pulsed flow, $\text{cm} \cdot \text{min}^{-1}$
k_s	mass transfer coefficient for steady flow, $\text{cm} \cdot \text{min}^{-1}$
M'_0	function of ξ_0 and Ψ_0 defined by Equation (6)
M'_{10}	frequency dependent parameter defined by Equation (33)
$M_0(x)$	modulus of zero order Bessel function with argument x
$M_1(x)$	modulus of 1st order Bessel function with argument x
p	pressure, psi
p_o	pressure on the permeate side, psi
Δp	transmembrane pressure, $(p - p_o)$, psi
Δp_0	initial transmembrane pressure, psi
P	dimensionless pressure, $2(p - p_o)/\rho u_0^2$

Pe_0	Peclet number based on initial permeation velocity, $v_{w0}r_i/D$
$\mathcal{P}$	power requirement per unit length of tube
$Q(t)$	instantaneous value of flowrate, $\text{cm}^3 \cdot \text{min}^{-1}$
Q_s	steady flow component of flow rate
Q_0	amplitude of oscillatory component of flowrate
$\bar{Q}$	time-averaged flowrate
r, R	radial direction; dimensionless radial direction, r/r_i
r_i	radius of tubular membrane module, cm
Re_{w0}	Reynolds number based on initial permeation velocity, $v_{w0}r_i/\nu$
t	time, min
u	axial velocity component in z-direction, $\text{cm} \cdot \text{min}^{-1}$
u'	oscillatory velocity component defined by Equation (5)
u_0	average velocity at channel inlet, $\text{cm} \cdot \text{min}^{-1}$
U	dimensionless axial velocity, u/u_0
v	radial velocity in r-direction, $\text{cm} \cdot \text{min}^{-1}$
v_{w0}	initial permeation velocity based on pure water flux, $\text{cm} \cdot \text{min}^{-1}$
V	dimensionless radial velocity, v/v_{w0}
z, Z	axial direction; dimensionless axial direction, $v_{w0}z/u_0r_i$
$Z_{\max}$	maximum length of membrane module, dimensionless

Greek Symbols:

β	solute rejection coefficient at the membrane surface
ε	oscillatory to steady flow pressure gradient amplitude ratio
ν	kinematic viscosity of feed solution, $\text{cm}^2 \cdot \text{min}^{-1}$
μ	dynamic viscosity, $\text{g} \cdot \text{cm}^{-1} \cdot \text{min}^{-1}$
ω	frequency, $\text{radian} \cdot \text{min}^{-1}$
Ω	dimensionless frequency parameter, $r_i(\omega/\nu)^{1/2}$
Φ_0	phase angle defined by Equation (7)
Φ_1	phase angle defined by Equation (35)
π	osmotic pressure of the solute in solution, psi
π_o	osmotic pressure of permeate, psi
$\Delta\pi$	transmembrane osmotic pressure, $(\pi - \pi_o)$, psi
Π	dimensionless osmotic pressure, $2(\pi - \pi_o)/\rho u_0^2$
Ψ_0	functions $\Theta_0(x)$ defined by Equation (7)
ρ	density of feed solution, $\text{g} \cdot \text{cm}^{-3}$
σ	phase angle defined by Equation (34)
τ	dimensionless time, tv_{w0}/r_i
$\Theta_0(x)$	phase of Bessel function of order zero with argument x
$\xi_0(x)$	function of $M_0(x)$ defined by Equation (7)

LITERATURE CITED

1. C.A. Smolders, and van den Boomgaard, Th., *J. Memb. Sci.*, **40**, 121 (1989).
2. R.L. Merson, and D.N. Lee, *J. Food Sci.*, **43**, 403 (1976).
3. M. Cheryan, *Ultrafiltration Hand Book*, Technomic Publishing Inc., Pa 17604 (1986).
4. R. Wallace, and R. Gable, *Polymer Eng. & Sci.*, **14**, 92 (1974).
5. H. Nomura, M. Seno, H. Takahashi, and T. Yamabe, *Desalination*, **29**, 239 (1979).

6. G. Belfort, and B. Marx, *Desalination*, **28**, 521 (1979).
7. J.A. Howell, and D. Velicangil, *J. Appl. Polymer Sci.*, **27**, 21 (1982).
8. H. Bauser, H. Chmiel, N. Stroh, and E. Walitza, *J. Memb. Sci.*, **11**, 321 (1982).
9. S. Bruin, A. Kikkert, J.A.G. Weldering, and J. Hiddnik, *Desalination*, **35**, 223 (1980).
10. S.T. Hwang, and K. Kammermeyer, *Membranes in Separations*, Wiley Interscience Publications, NY (1975).
11. D.G. Thomas, W.L. Griffith, and R.M. Keller, *Desalination*, **9**, 33 (1971).
12. P. Dejmek, B. Funeteg, B. Halstrom, and L. Wingle, *J. Food Sci.*, **39**, 403 (1974).
13. A.L. Copas, and S. Middleman, *Ind. Eng. Chem. Proc. Des. Dev.*, **13**, 143 (1974).
14. J. Hiddink, D. Kloosterboer, and S. Bruin, *Desalination*, **35**, 149 (1980).
15. E. Lowe, E.L. Durkee, R.L. Merson, K. Ijichi, and S.L. Cimino, *Food Technology*, **23**, 753 (1969).
16. E. Lowe, and E.L. Durkee, *J. Food Sci.*, **36**, 31 (1971).
17. K. Ramanadhan, and W.N. Gill, *AIChE J.*, **15**, 872 (1971).
18. S.P. Muhlenkamp, and S. Srinivasan, *Proc. 4th Intl. Symp. Fresh Water from the Sea*, **4**, 251 (1973).
19. G.H. Robertson, J.J. Olieman, R.A. Carlson, and D.F. Farkas, *AIChE Symp. Ser.*, **78** (218): 129 (1982).
20. H. Lee, and E.N. Lightfoot, *AIChE J.*, **20**, 335 (1974).
21. G.M. Rios, H. Rakotoarisoa and T. de la Fuente, *J. Memb. Sci.*, **34**, 331 (1987).
22. G.M. Rios, H. Rakotoarisoa and T. de la Fuente, *J. Memb. Sci.*, **35**, 147 (1988).
23. B. Hallstrom, and M.L. Levia, *Desalination*, **24**, 273 (1978).
24. S. Takdono, H. Iwahori, T. Yabushita, Y. Imaura, H. Ishizuka, and S. Tamaru, *Desalination*, **49**, 347 (1984).
25. K.H. Kroner, and V. Nissinen, *J. Memb. Sci.*, **36**, 85 (1988).
26. T.J. Kennedy, R.L. Merson, and B.J. McCoy, *Chem. Eng. Sci.*, **29**, 1927 (1974).
27. G. Belfort, *J. Memb. Sci.*, **40**, 123 (1990).
28. R.B. Bird, W.E. Stewart, and E.N. Lightfoot, *Transport Phenomena*, John Wiley, NY (1960).
29. M.F. Edwards, and W.L. Wilkinson, *Trans. Instn. Chem. Engrs.*, **49**, 85 (1971).
30. J.R. Womersley, *J. Physiol. Lond.*, **127**, 553 (1955).
31. S. Ilias, and R. Govind, *J. Memb. Sci.*, **39**, 125 (1988).
32. U. Merten, *Ind. Eng. Chem. Fund.*, **2**, 229 (1963).
33. M.H.I. Baird, G.J. Duncan, J.I. Smith, and J. Taylor, *Chem. Eng. Sci.*, **21**, 197 (1966).